Introduction

Our work bridges the concepts of Quantum Computing and Digital Signal Processing (DSP). The presented algorithm transforms a qubit representation into a discrete time signal through the discrete Fourier Transform. Through this algorithm, the Kronecker product of two qubits can be performed by the circular convolution of two real discrete time signals.

This approach aims to integrate quantum computing principles into DSP systems to enhance the computational capabilities of current devices.

Methods

The DSP computes an algorithm that involves a spectral expansion and reduction to establish an isomorphism on the representation of qubits and discrete time signals.

Each N-qubit register $Q^N$ is represented as a signal in the frequency domain. The signal is normalized to unit energy:

$$S^K[k] = \begin{cases} 
0, & k = 0, \\
\frac{1}{L} Q^N[k], & 0 < k < \frac{K}{2}, \\
0, & k = \frac{K}{2}, \\
Q^N[K - k]^*, & \frac{K}{2} < k < K.
\end{cases}$$

The frequency spectrum of $S$ is zero-padded to align with the conditions set for the transformation ($K=2N+2$).

The signal is obtained in the time domain by the inverse discrete Fourier transform.

The interpolation and decimation techniques employed in the transformation can be implemented by DSP operations.

The merging of two qubits into a single entity is achieved through the Kronecker product. For a pair, each qubit is a unit vector in a 2-dimensional complex vector space. The Kronecker product combines these qubits, creating a higher-dimensional space with the following order of presentation for a Qubit register $Q^M (M=N^2)$:

$$\begin{bmatrix} 
\alpha_1 \alpha_0 \\
\beta_1 \beta_0 \\
\beta_1 \alpha_0 \\
\beta_1 \beta_1
\end{bmatrix} = \begin{bmatrix} 
\alpha_1 \\
\beta_1 \\
\alpha_0 \\
\beta_0
\end{bmatrix} \otimes \begin{bmatrix} 
\alpha_0 \\
\beta_0
\end{bmatrix}$$

Algorithmic Construction

To match the exhibition order of the Kronecker product, the developed algorithm set one signal as the most significant one and the other as the least significant one ($K=10, N=2$).

$$S_i^N[k] = [0; \alpha_i; \alpha_i; \beta_i; \beta_i; 0; \beta_i^*; \alpha_i^*]$$

$$S_0^N[k] = [0; \alpha_0; \beta_0; \alpha_0; \beta_0; 0; \beta_0^*; \alpha_0^*]$$

Such that the frequency spectrum of $S_i$ is aligned with the conditions set for the transformation ($G=2M+2$).

The convolution in the time domain corresponds to the point wise multiplication in the frequency domain. The resulting signal is the spectral expansion of the Kronecker product of two qubits, where $L$ is a normalization factor.

$$L \cdot K^G[g] = [0; \alpha_1 \cdot \alpha_0; \beta_1 \cdot \alpha_0; \beta_1 \cdot \beta_0; 0; \beta_1^* \cdot \alpha_0^*; \beta_1^* \cdot \beta_0^*; \alpha_1^* \cdot \alpha_0^*; \alpha_1^* \cdot \beta_0^*]$$

Conclusions

This poster presents a technique for representing a qubit’s register as a real discrete-time signal. This technique’s scalability enables the execution of advanced computations on existing devices using a digital signal processing unit.

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